A PROPOSED JOURNAL OF APPLIED MATHEMATICS

During the past two years a school for advanced instruction in applied mechanics has been conducted at Brown University with notable success. One result has been to form a group of men from many different centers who desire to make a permanent place for advanced applied mathematics in this country. To this end, it has now been decided to publish a *Journal of Applied Mathematics*, to appear quarterly, and to form a volume of 300–400 pages annually. The first issue will be as for January, 1943, although actual publication may be delayed beyond that month.

From time to time over the past two decades, a journal of this type has been proposed, but the project has never been brought to a focus. This journal, as now planned, is to fill the gap between purely engineering journals and purely mathematical ones. It will provide a place where the various subjects can be regarded as sciences, and where any piece of work which fills a gap in this structure or extends it, can be accepted without having to show immediate application to engineers or fundamentally new ideas of mathematical interest to mathematicians.

Not only must applied mathematics serve the war effort, as it is now doing in increasing measure, but, looking ahead to the post-war period, it appears essential that America should be strong in the whole field of Science in order that it serve as a world center for advanced instruction and research. The new journal will aid in bringing America to the forefront in applied mechanics and in other branches of applied mathematics. It is hoped that many mathematicians will approve and will give this project both moral and financial support.

The Board of Editors will be:

Dr. H. L. Dryden, Physicist, National Bureau of Standards.
Dr. T. C. Fry, Mathematical Research Director, Bell Telephone Laboratories.
Dr. W. Prager, Professor of Applied Mathematics, Brown Univ.
Dr. I. S. Sokolnikoff, Professor of Mathematics, Univ. of Wisconsin.
Dr. J. L. Synge, Professor of Applied Mathematics, Univ. of Toronto.
Dr. Th. von Kármán, Professor of Aeronautical Engineering, Calif. Inst. of Tech.; Consultant, Northrop Aircraft, Inc.

These names will carry a message of confidence of high standards to all who know the field. Some fifteen others, eminent in their fields, will be asked to accept appointments as collaborators.

E. R. HEDRICK

OCCUPATIONAL CLASSIFICATION OF MATHEMATICIANS

General Lewis B. Hershey, Director of Selective Service, has issued Occupational Bulletin No. 23 which certifies that the educational services of instructors and professors of mathematics in colleges, universities, or professional schools are activities essential to the war effort. This Bulletin supplements Occupational Bulletin No. 10 which was distributed to department chairmen by the Secretary's office on June 30, 1942. Bulletin No. 10 and Bulletin No. 23 are given at the end of this note.

This memorandum is published with the realization that the whole situation with respect to our colleges and universities will undoubtedly be fundamentally altered by the legislation concerned with the lowering of the draft age. Nevertheless, we do believe that plans will be developed whereby the government will use the machinery already existing in our colleges and universities in order to train young men in the mathematics and science which are the necessary equipment of Army and Navy officers. The need for instructors in mathematics is bound to increase as the tempo of the war is increased.

The Secretary of War has stated that teachers in fields that are essential to the war effort are doing the job that the country wants them to do and are performing their full duty in the war effort. With this in mind, every effort should be made to conserve the supply of mathematicians; this supply is already greatly depleted. Mathematicians should not be discouraged from assuming positions in fields other than teaching, if these offer greater opportunities for service to the nation in the war effort. On the other hand, mathematics teachers should not feel that their services will shortly be unnecessary and consequently rush into positions where their contribution is less vital than that of teaching men who are preparing for commissions in the armed forces. Mathematics at the college level will have to be taught to prospective officers. We hope that this teaching will be done in our colleges and universities. Whether it is done there or in schools set up by the government, mathematicians will be needed and the supply should be conserved.

I. Requests for Occupational Classification

In view of the recent Bulletin, we wish to offer the following suggestions for securing the proper classification of mathematicians:

(a) *For those who have completed their training, i.e., instructors and men of professorial grade.* Requests for continued occupational de-

ferment or for reclassification should be based on Occupational Bulletin No. 23 in which the Manpower Commission has certified as essential to the war effort professors and instructors engaged in full-time instruction and research in mathematics. This Bulletin should be attached to every request. In addition, Occupational Bulletin No. 10, which certifies that there is a serious shortage of mathematicians, should be attached. Local boards must be supplied with all the necessary information; this should include evidence that the registrant cannot be replaced. The request for reclassification should be made on Form 42A which can be obtained from the registrant's local board.

(b) *For those in training, i.e., graduate assistants and undergraduate majors in mathematics.* The officer requesting the deferment should file Form 42A directly with the local board with the ordinary questionnaire filled out by all registrants. Reference should be made to Occupational Bulletin No. 10. It is urged that a copy of this Bulletin be attached to Form 42A when it is filed.

II. Appeals

Having made request for occupational deferment, the person making the request should be prepared to exert every effort to secure that deferment, including the making of an appeal. The appeal may be made by the registrant, his employer, or the government appeal agent. When it is clear that the end of negotiations with the local board has been reached, without having modified the registrant's 1A classification, the appeal should be carried through. *Such an appeal must be made within ten days of the date when the local board mailed to the registrant a notice of his classification.* The form for appeals appears at the end of the questionnaire.

III. Additional Steps

National Roster of Scientific and Specialized Personnel,
10th and U Streets, N. W., Washington, D. C.

Any mathematician who has not already done so should register immediately with the National Roster. This may be done by writing a letter to the Roster at the above address, and asking for appropriate forms in mathematics. In this communication, the writer should indicate whether he is subject to the Selective Training and Service Act of 1940.

The National Roster may take appropriate action in the case of professional mathematicians and graduate students in mathematics.

Under a cooperative plan with the Selective Service System, the Roster may send to the National Headquarters of the Selective Service System appropriate information about professionally trained men of military age, and that office in turn may then forward letters about these men through the various State Directors to the individual local boards, in order to assist in proper classification.

It is essential that a man (or his employing institution) should *not* wait until he is placed in Class 1A before writing to the Roster. The Roster should be kept informed as to any changes in his Selective Service status or in his employment. In communicating with the Roster, a man should give the following information: (1) the name, number, and address of his Selective Service local board; (2) his own order number; (3) a description of his specific duties, especially as related to the war effort.

Office of Scientific Personnel, National Research Council,
2101 Constitution Avenue, Washington, D. C.

Dean Homer L. Dodge is now Director of the Office of Scientific Personnel. In cases where an appeal has been made and developments indicate that a proper decision may not be reached, it is advisable to contact Dean Dodge. He must be informed as to the applicant's order number and the name, number, and address of the local board under which he is registered. Dean Dodge has offered to consult with the State Director concerned or with Selective Service Headquarters as the urgency of the case demands.

However, the Office of Scientific Personnel must start where the person making the original request leaves off and can do little to make up for deficiencies in the presentation of the case by the employer. Unless there is presented evidence that a genuine, but unsuccessful, attempt has been made to secure a replacement for the particular registrant in question, it will be impossible for the Office of Scientific Personnel to be of assistance.

Copies of Occupational Bulletin No. 10 and of Occupational Bulletin No. 23 will be furnished, on request, by the office of the Secretary.

MARSTON MORSE, *President*
J. R. KLINE, *Secretary*

November 2, 1942.

National Headquarters

SELECTIVE SERVICE SYSTEM

21st Street and C Street, N. W.
Washington, D. C.

COPY September 30, 1942

OCCUPATIONAL BULLETIN NO. 23

EFFECTIVE: IMMEDIATELY
SUBJECT: EDUCATIONAL SERVICES

1. The War Manpower Commission has certified that educational services are essential to the support of the war effort.

2. This bulletin covers the following essential activities which are considered as included within the list attached to Local Board Release No. 115, as amended:

(a) *Educational services*: Public and private industrial vocational training; elementary, secondary and preparatory schools; junior colleges, colleges, universities and professional schools; educational and scientific research agencies; and the production of technical and vocational training films.

3. In considering registrants engaged in educational services there must be taken into consideration the following:

(a) The kind of institution in which the registrant is engaged;

(b) the occupation of the registrant in that institution; and

(c) the classroom studies under the registrant's instruction, supervision, or administration jurisdiction. Attached is a list of occupations by institutions and classroom studies in educational services which require a reasonable degree of training, qualification, or skill to perform the duties involved. It is the purpose of this list to set forth by institutions and classroom studies the important occupations in educational services which must be filled by persons capable of performing the duties involved in order that the essential portions of the activity may be maintained. Item 4 of the list does not include classroom studies but occupations which shall be considered in the same manner as any other occupations. The entire list is confined to occupations which require more than six months of training and preparation.

4. In classifying registrants employed in these activities, consideration should be given to the following:

(a) The training, qualification, or skill required for the proper discharge of the duties involved in his occupation;

(b) the training, qualification, or skill of the registrant to engage in his occupation; and

(c) the availability of persons with his qualifications or skill, or who can be trained to his qualification, to replace the registrant and the time in which such replacement can be made.

LEWIS B. HERSHEY, *Director*

CRITICAL OCCUPATIONS

Educational Services

1. ELEMENTARY, SECONDARY AND PREPARATORY SCHOOLS

   (a) Superintendents of elementary, secondary and preparatory school systems; and
   (b) teachers who are engaged in full-time instruction in one or more of the following subjects:

Aeronautics	Mathematics
Biology	Physics
Chemistry	Radio

2. JUNIOR COLLEGES, COLLEGES, UNIVERSITIES AND PROFESSIONAL SCHOOLS, EDUCATIONAL AND SCIENTIFIC RESEARCH AGENCIES

   (a) Presidents, Deans, and Registrars in junior colleges, colleges, universities and professional schools; and
   (b) professors and instructors engaged in full-time instruction and research in one or more of the following subjects:

Agricultural Sciences	Engineering Sciences	Navigation, Aerial and Marine
Architecture, Naval	Geology	
Astronomy	Industrial Management	Oceanography
Bacteriology	Mathematics	Pharmacy
Biology	Medicine and Surgery	Physics
Chemistry	Metallurgy	Physiology
Dentistry	Meteorology	Veterinary Sciences

3. PUBLIC AND PRIVATE INDUSTRIAL VOCATIONAL TRAINING

   (a) Superintendents of public and private industrial vocational training; and
   (b) teachers who are engaged in full-time instruction in one or more of the following subjects designated to prepare students for war activities:
   Trade, Vocational and Agricultural subjects (such as, Machine Shop practice, Aircraft, Sheetmetal Work, and similar subjects) and in Vocational Rehabilitation.

4. PRODUCTION OF TECHNICAL AND VOCATIONAL TRAINING FILMS

   (a) Persons engaged full-time and exclusively in the production of technical and vocational training films for the Army, Navy and war production industries.

Animator	Project Supervisor
Cameraman	Technical Consultant
Cutter	Technical Writer
Film Editor	Sound Engineer

National Headquarters
SELECTIVE SERVICE SYSTEM
21st Street and C Street, N. W.
Washington, D. C.

COPY

June 18, 1942

OCCUPATIONAL BULLETIN (NO. 10)

SUBJECT: SCIENTIFIC AND SPECIALIZED PERSONNEL
EFFECTIVE: IMMEDIATELY
DISTRIBUTION: STATE DIRECTORS
BOARD OF APPEAL MEMBERS
LOCAL BOARD MEMBERS
GOVERNMENT APPEAL AGENTS

PART I

1. There are certain persons trained, qualified, or skilled in scientific and specialized fields who, if engaged in the practice of their respective professions, are in a position to perform a vital service in activities necessary to war production and in activities essential to the support of the war effort.

PART II

1. The National Roster of Scientific and Specialized Personnel has certified to the Director of Selective Service that in activities necessary to war production and in activities essential to the support of the war effort, there are certain "critical occupations" which for the proper discharge of the duties involved require a high degree of training, qualification, or skill in scientific and specialized fields. The critical occupations in these scientific and specialized fields, as certified to the Director of Selective Service, are listed on page 4 attached to this bulletin.

2. All of these critical occupations, as listed, require highly specialized periods of training of two years or more. The critical occupations on the attached list exist within the provisions of Part V, Memorandum to All State Directors (I-405).

PART III

1. The National Roster of Scientific and Specialized Personnel has certified to the Director of Selective Service that there are serious shortages of persons trained, qualified, or skilled to engage in these critical occupations in activities necessary to war production and in activities essential to the support of the war effort. These shortages exist within the provisions of Part VII, Memorandum to All State Directors (I-405), and accordingly careful consideration for occupational classification should be given to all persons trained, qualified, or skilled in these critical occupations and who are engaged in activities necessary to war production or essential to the support of the war effort.

PART IV

1. There are many registrants who are in training and preparation to acquire the qualification or skill to engaged in these critical occupations. Normally the period of training and preparation to acquire the necessary qualification or skill in these scientific and specialized fields extends over a period of four academic years in a recognized academic, professional, or technical college or university. In many instances, however,

it is necessary for persons to have additional study in a recognized academic, professional, or technical college or university in order to acquire the more highly specialized qualification or skill necessary for the performance of particular services in activities necessary to war production or essential to the support of the war effort. Persons engaging in further studies in addition to the four academic years normally required are referred to as graduate or postgraduate students.

2. A registrant who is in training and preparation for one of these scientific and specialized fields may not be considered for occupational classification until the close, or approximately the close, of his second or sophomore year in a recognized college or university.

3. A registrant who is in training and preparation for one of these scientific and specialized fields may be considered for occupational deferment at the close, or approximately at the close, of his second or sophomore year in a recognized college or university if he is pursuing a course of study upon the successful completion of which he will have acquired the necessary training, qualification, or skill, and if he gives promise of continuing and will be acceptable for continuing such course of study and will undertake actual further classroom work within a period of not to exceed four months from the close of his second year.

4. A registrant who is in training and preparation for one of these scientific and specialized fields shall be considered for occupational classification during his third and fourth years in a recognized college or university, provided that he gives promise of the successful completion of such course of study and the acquiring of the necessary degree of training, qualification, or skill.

5. A graduate or postgraduate student who is undertaking further studies for these scientific and specialized fields, following the completion of the normal four academic years, may be considered for occupational classification if, in addition to pursuing the additional studies, he is also acting as "graduate assistant" in a recognized college or university or is engaged in scientific research related to the war effort and which is supervised by a recognized Federal agency. A graduate assistant is a student in postgraduate studies who, in addition, is engaged in the teaching and instruction of undergraduate students in these scientific and specialized fields.

6. When a registrant has completed his training and preparation in a recognized college or university and has acquired a high degree of training, qualification, or skill in one of these scientific and specialized fields, such registrant should then be given the opportunity to become engaged in the practice of his profession in an activity necessary to war production or essential to the support of the war effort. In many instances following graduation from a recognized college or university, a certain period of time will be required in the placing of trained, qualified, or skilled personnel in an essential activity. When a registrant has been deferred as a necessary man in order to complete his training and preparation, it is only logical that his deferment should continue until he has an opportunity to use his scientific and specialized training to the best interest of the nation. Accordingly, following graduation from a recognized college or university in any of these scientific and specialized fields, a registrant should be considered for further occupational classification for a period of not to exceed 60 days in order that he may have an opportunity to engage in a critical occupation in an activity necessary to war production or essential to the support of the war effort, provided that during such period the registrant is making an honest and diligent effort to become so engaged.

LEWIS B. HERSHEY, *Director*

CRITICAL OCCUPATIONS

SCIENTIFIC AND SPECIALIZED PERSONNEL

- Accountants
- Chemists
- Economists
- Engineers:
  - Aeronautical Engineers
  - Automotive Engineers
  - Chemical Engineers
  - Civil Engineers
  - Electrical Engineers
  - Heating, Ventilating, Refrigerating, and Air Conditioning Engineers
  - Marine Engineers
  - Mechanical Engineers
  - Mining and Metallurgical Engineers, including Mineral Technologists
  - Radio Engineers
  - Safety Engineers
  - Transportation Engineers—Air, Highway, Railroad, Water
- Geophysicists
- Industrial Managers
- Mathematicians
- Meteorologists
- Naval Architects
- Personnel Administrators
- Physicists, including Astronomers
- Psychologists
- Statisticians

THE SUMMER MEETING IN POUGHKEEPSIE

The forty-eighth Summer Meeting of the Society and the twenty-fourth Colloquium were held at Vassar College, Tuesday to Thursday, September 8–10, 1942. The Mathematical Association of America met on Monday morning and afternoon. The Institute of Mathematical Statistics met Wednesday morning and jointly with the Society Tuesday afternoon and Wednesday afternoon. About three hundred persons attended the meetings among whom were the following one hundred ninety-eight members of the Society:

C. R. Adams, R. B. Adams, R. P. Agnew, Warren Ambrose, R. C. Archibald, K. J. Arnold, L. A. Aroian, K. J. Arrow, W. L. Ayres, F. E. Baker, G. M. Bareis, Walter Bartky, Stefan Bergman, Felix Bernstein, Henry Blumberg, J. W. Bower, R. W. Brink, A. B. Brown, J. A. Bullard, F. J. H. Burkett, Hobart Bushey, J. H. Bushey, S. S. Cairns, W. D. Cairns, B. H. Camp, W. B. Carver, W. F. Cheney, D. E. Christie, J. A. Clarkson, L. W. Cohen, Nancy Cole, Esther Comegys, T. F. Cope, A. H. Copeland, Richard Courant, C. C. Craig, J. H. Curtiss, D. R. Davis, F. F. Decker, R. P. Dilworth, J. L. Doob, M. E. Elveback, Benjamin Epstein, Paul Erdös, W. K. Feller, M. M. Flood, L. R. Ford, R. M. Foster, K. O. Friedrichs, R. E. Fullerton, Abe Gelbart, B. P. Gill, R. E. Gilman, Michael Goldberg, Cornelius Gouwens, H. S. Grant, R. E. Greenwood, T. N. E. Greville, C. C. Grove, V. G. Grove, E. J. Gumbel, Theodore Hailperin, D. W. Hall, O. G. Harrold, O. C. Hazlett, C. M. Hebbert, G. A. Hedlund, E. R. Hedrick, Edward Helly, L. S. Hill, Einar Hille, T. R. Hollcroft, M. W. Hopkins, G. M. Hopper, Harold Hotelling, S. E. Hotelling, E. M. Hull, C. C. Hurd, Witold Hurewicz, W. A. Hurwitz, Dunham Jackson, Fritz John, R. A. Johnson, A. W. Jones, B. W. Jones, F. B. Jones, H. A. Jordan, Wilfred Kaplan, Irving Kaplansky, Edward Kasner, S. Y. Keyser, B. F. Kimball, J. R. Kline, A. E. Landry, R. E. Langer, G. A. Larew, Solomon Lefschetz, Charles Loewner, C. I. Lubin, Eugene Lukacs, N. H. McCoy, C. C. MacDuffee, Saunders MacLane, H. F. MacNeish, H. B. Mann, Morris Marden, A. J. Maria, D. H. Maria, M. H. Martin, M. P. Martin, A. E. Meder, F. H. Miller, E. B. Mode, E. C. Molina, Deane Montgomery, C. N. Moore, E. M. Morenus, Valdimir Morkovin, Richard Morris, D. S. Morse, Marston Morse, M. E. Munroe, Otto Neugebauer, A. V. Newton, K. L. Nielsen, R. E. O'Connor, E. G. Olds, Oystein Ore, Gordon Pall, C. R. Phelps, H. B. Phillips, Everett Pitcher, Hillel Poritsky, G. B. Price, M. H. Protter, A. L. Putnam, S. E. Rasor, L. L. Rauch, Mina Rees, C. F. Rehberg, W. T. Reid, R. G. D. Richardson, J. F. Ritt, M. S. Robertson, H. A. Robinson, S. L. Robinson, R. E. Root, R. A. Rosenbaum, J. H. Rosenbloom, P. C. Rosenbloom, Raphael Salem, Hans Samelson, Arthur Sard, F. E. Satterthwaite, L. J. Savage, Henry Scheffé, Abraham Schwartz, G. E. Schweigert, R. G. Simond, A. J. Smith, Gertrude Smith, Ernst Snapper, Virgil Snyder, D. E. Spencer, V. E. Spencer, Abraham Spitzbart, A. H. Sprague, H. W. Steinhaus, J. J. Stoker, R. R. Stoll, A. H. Stone, S. R. R. Struik, Otto Szász, Gabor Szegö, J. D. Tamarkin, C. J. Thorne, A. W. Tucker, J. W. Tukey, D. F. Votaw, Abraham Wald, R. M. Walter, Louis Weisner, B. A. Welch, M. E. Wells, E. T. Welmers, George Whaples, H. S. White, G. T. Whyburn, R. L. Wilder, S. S. Wilks, W. L. G. Williams, E. W. Wilson, Jack Wolfe, Jacob Wolfowitz, E. R. Worthington, C. H. Yeaton, J. W. T. Youngs, Antoni Zygmund.

The Colloquium Lectures on the subject *Topology of manifolds* were delivered by Professor R. L. Wilder of the University of Michigan on Tuesday morning and afternoon and Wednesday and Thursday mornings. The presiding officers at the four lectures were, in order, President Marston Morse, Professor Solomon Lefschetz, Professor G. T. Whyburn, and Vice President J. D. Tamarkin.

There was a short business meeting Wednesday morning at the beginning of the general session, President Marston Morse presiding. Secretary J. R. Kline reported briefly concerning actions of the Council and President Morse spoke about the present status with respect to military service of teachers and students of mathematics. Lieutenant C. C. Hurd described the training program at the United States Coast Guard Academy at New London, Connecticut.

On Wednesday at 2 P.M., Professor W. L. Ayres of Purdue University gave an address entitled *Transformations with periodic properties*. At 3:15 P.M., a symposium on *The applicability of mathematical statistics to war efforts*, was held jointly with the Institute of Mathematical Statistics. Addresses were given on *Statistical prediction with special reference to the problem of tolerance limits* by Professor S. S. Wilks of Princeton University, and *On the nature of mathematical statistics in quality control* by Dr. W. E. Deming of the U. S. Department of Commerce. The discussion leaders were Professor J. H. Curtiss of Cornell University, and Dean Walter Bartky of the University of Chicago.

The presiding officers for the sessions of short papers were: Analysis, Tuesday morning, Professor R. E. Langer; Algebra, Geometry, and Topology, Tuesday morning, Professor A. W. Tucker; Mathematical Statistics (a joint session with the Institute of Mathematical Statistics), Tuesday afternoon, Professor C. C. Craig; Topology, Thursday morning, Professor R. L. Wilder; Analysis and Applied Mathematics, Thursday morning, Professor Gabor Szegö. Professor J. R. Kline presided at the general session at 2 P.M., Wednesday and Lieutenant Colonel L. E. Simon at the symposium held jointly with the Institute of Mathematical Statistics.

Headquarters for the meeting were in Main Building. Rooms in this building were made available to members of the three organizations and their guests.

On Wednesday evening in Main Building there was a dinner for members of the Society, the Association, and the Institute. Professor H. S. White was toastmaster. President H. N. MacCracken of Vassar College gave an address of welcome. Professor R. E. Langer spoke of the impetus being given to mathematics in our secondary

schools and colleges. Professor W. T. Reid presented a resolution of thanks to the President, the Department of Mathematics, the Local Committee, and everyone concerned at Vassar College, for the competent arrangements and cordial hospitality. The resolution was adopted unanimously. Following the dinner members of the Department of Music of Vassar College gave an excellent concert in Belle Skinner Hall.

On Monday afternoon in Ely Hall, a tea was given for members of the three organizations and their guests by the Department of Mathematics of Vassar College.

The Council met on Tuesday, September 8, at 8:00 P.M. in Main Building at Vassar College.

The Secretary announced the election of the following twenty persons to membership in the Society:

Dr. William D. Berg, State University of Iowa;
Mr. Howard Jesse Eckweiler, New York University;
Mr Joseph Owen Evans, Shreveport Aeronautical Institute, Shreveport, La.;
Mother Ethelyn Fox, Barat College of the Sacred Heart, Lake Forest, Ill.;
Mr. William J. Fry, Naval Research Laboratory, Washington, D. C.;
Mr. Cecil Hastings, Jr., University of Florida;
Mr. Cyrus Giles Hill, J. G. Wray and Co., Engineers, Chicago. Ill.;
Professor Truman Lee Kelley, Graduate School of Education, Harvard University;
President Eugene McDermott, Geophysical Service, Inc., Dallas, Tex.;
Dr. Leonard McFadden, Virginia Polytechnic Institute;
Mr. Frank Albert McMahon, Brooklyn College Evening Session;
Dr. Henry Malin, College of the City of New York;
Mr. William Edwin Manring, American Cyanamid and Chemical Corporation, New York, N. Y.;
Professor Philip B. Norman, Tusculum College, Greenville, Tenn.;
Professor Edward Russell Stabler, Hofstra College, Hempstead, N. Y.;
Dr. Domina Eberle Spencer, Massachusetts Institute of Technology;
Dr. Robert L. Swain, University of Wisconsin;
Mr. Hsio-Fu Tuan, Princeton University;
Mr. David Freeman Votaw, Jr., Princeton University;
Mr. Hyman Joseph Zimmerberg, University of Chicago.

It was also announced that Texas Technological College, Lubbock, Texas, had been elected to institutional contributing membership.

Professor Michel Plancherel of the Ecole Polytechnique Fédérale, Zurich, Switzerland, was admitted to the Society in accordance with the reciprocity agreement with the Swiss Mathematical Society.

The following appointments by President Marston Morse were reported: as representative of the Society at the Inaugural Ceremony of the Polish Institute of Arts and Sciences in America on May 15, 1942, Professor C. C. MacDuffee; as representative of the Society at the inauguration of Harry Noble Wright as President of The College

of the City of New York on September 30, 1942, Professor C. C. MacDuffee; as a member of the Committee on Places of Meetings, to replace Professor P. R. Rider (term to expire December 31, 1944), Professor E. W. Chittenden.

It was announced that reports were sent in August to the Carnegie Corporation and Rockefeller Foundation regarding their grants to Mathematical Reviews. The Secretary also announced that the problem of the importation of journals from European Axis and Axis-controlled countries had been solved for Mathematical Reviews. Under an arrangement concluded with the American Library Association, Mathematical Reviews is permitted to purchase all the journals to which it has been subscribing, thus continuing full coverage of the mathematical literature of the scientific world.

Certain invitations to give invited addresses were announced: Professor J. L. Synge for the October, 1942, meeting in New York; Professors Harry Bateman, L. M. K. Boelter, J. Holmboe, and H. U. Sverdrup for the November, 1942, meeting in Los Angeles; Professor Deane Montgomery for the 1942 Annual Meeting in New York; and Professor Emil Artin for the April, 1943, meeting in Chicago.

On recommendation of the Committee on Places of Meetings (Professor A. B. Coble, chairman), the Annual Meeting of 1943 was set at Cleveland, Ohio, in connection with the meetings of the American Association for the Advancement of Science. Times and places of meetings during 1943 were set as follows: in New York City on February 27; in New York City on April 23–24; in Chicago on April 23–24; at Stanford University on April 24; in Corvallis, Oregon, on June 16–17 (with A.A.A.S.); in New York City on October 30.

Because of travel difficulties and other problems created by the war, the Council voted to withdraw its acceptance of the invitation to meet at the University of Colorado for the Summer Meeting of 1943. It is the intention of the Society to meet at Colorado as soon as conditions permit. Arrangements are under way for holding a summer meeting at a more centrally-located institution.

The eighteenth Josiah Willard Gibbs Lecture is to be given by Professor Harry Bateman at the Annual Meeting of 1943 in Cleveland, Ohio.

On recommendation of the Colloquium Editorial Committee, the Council voted to invite Professor Einar Hille to deliver a series of Colloquium Lectures in 1944.

Professor Marston Morse was appointed representative on the National Research Council for a period of three years beginning July, 1943.

The Secretary announced the provisions of the will of the late Dr. Robert Henderson who was, for many years, a Trustee of the Society and deeply interested in all of its activities. The Society will share the income of his estate with two other beneficiaries, and, upon the death of the other legatees, the Society will receive the full principal of the estate for the Endowment Fund.

President Morse appointed the following committees: Committee on the Award of the Bôcher Prize, to be awarded at the Annual Meeting of 1943, for papers in Analysis published during the period 1938–1942, Professors T. H. Hildebrandt (chairman), Philip Franklin, and John von Neumann; Committee to Select Gibbs Lecturers for 1944 and 1945, Professors J. H. Van Vleck (chairman), B. O. Koopman, and E. J. McShane.

The Council voted to continue the practice, inaugurated two years ago, of presenting on the ballot twice as many candidates for vice-president and for members-at-large of the Council as there are positions to be filled. It was also voted that the ballot continue to have a place where members may present suggestions for trustees, officers, and members of the Council, these suggestions to be considered by the Nominating Committee for the following year. Members are urged to make more use of this method of placing names before the Nominating Committee.

Titles and cross references to the abstracts of papers read are given below. The papers were read as follows: papers 1–6 in the section for Analysis on Tuesday morning; papers 7–12 in the section for Algebra, Geometry, and Topology on Tuesday morning; papers 13–22 in the section for Mathematical Statistics on Tuesday afternoon; papers 23–27 in the general session Wednesday morning; papers 28–31 in the section for Topology Thursday morning; papers 32–34 in the section for Analysis and Applied Mathematics Thursday morning; and papers 35–89, whose abstract numbers are followed by the letter *t*, were read by title. Dr. Spencer was introduced by Professor D. J. Struik, Mr. Kincaid by Dr. D. W. Hall, Miss Marth by Professor Francis Regan, and Dr. Schwerdtfeger by Dr. Peter Scherk. Paper 2 was presented by Professor Clarkson, paper 3 by Professor Adams, paper 5 by Dr. Salem, paper 9 by Professor MacLane, paper 17 by Dr. Mann, paper 21 by Dr. Struik, paper 23 in the absence of Professor Whitney by Professor Saunders MacLane, paper 24 by Professor Hille, paper 30 by Dr. Samelson, and paper 33 by Dr. Poritsky.

1. M. E. Munroe: *On the finite dimensionality of certain Banach spaces.* (Abstract 48-7-241).

2. J. A. Clarkson and Paul Erdös: *On the approximation of continuous functions by polynomials.* (Abstract 48-9-258.)

3. C. R. Adams and A. P. Morse: *On approximating certain integrals by sums.* (Abstract 48-9-255.)

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BOOK REVIEWS

What is Mathematics? By Richard Courant and Herbert Robbins. London, New York and Toronto, Oxford University Press, 1941. 19+521 pp. $5.00.

Professor Courant is too expert a hand at mathematics to attempt a formal definition of the subject. Instead, he and his collaborator offer "an elementary approach to ideas and methods," from which the reader who persists to the end may draw his own conclusions. It will be interesting to American teachers and students to forecast what a few of those conclusions may be.

First, as to the material presented. Roughly, the topics treated cover the usual superior undergraduate course in mathematics through the calculus, with glimpses of such things as Riemannian geometry and Plateau's problem beyond the customary curriculum. It is a misapprehension, under which some authors not acquainted at first hand with mathematics as taught in the better colleges labor, that mathematics for the American undergraduate ends with the calculus, or at farthest with an introduction to differential equations. It does not; quite detailed courses in the theory of algebraic equations, the theory of numbers, projective geometry, both synthetic and analytic, modern plane elementary geometry, also occasionally non-Euclidean geometry, are offered and taken; and in some of the undergraduate schools of the larger universities a course in the theory of functions of a complex variable is a commonplace, as also is a thorough introduction to mechanics. So the average student or graduate of the better college courses in mathematics will not be wholly unprepared to appreciate the numerous illuminating sidelights which this book offers on what he already knows; nor will he be entirely blind to the attractions of things which he may see here for the first time. All this sums up to the opinion that the book is one for inspirational collateral reading, rather than a detailed manual for the mastery of any one of the topics it treats. The hypothetical layman who remembers a little of what he learned in college ten to fifty years ago will find the book both stimulating and demanding.

In the method of presentation, at least, there is something new and interesting for nearly every topic, if only a brief note calling attention to significant progress made within the past decade, or an excellent drawing making textual comment all but superfluous. A sound pedagogical strategy ensnares the reader at the outset (pp. 1–51) in the serenely useless properties of the common whole numbers,

catching his attention with simple but singularly recondite problems, solved and unsolved, while painlessly indoctrinating him with such useful things as the binomial theorem and geometrical progressions. The number system of analysis is then developed, culminating (p. 72) in the Dedekind cut as an alternative to the method of nested intervals previously developed in the text. After a brief excursus on the basic principle of analytic geometry, the discussion of the continuum is resumed in "the mathematical analysis of infinity," where the paradoxes and the present disorderly state of the foundations of mathematics are fairly and fearlessly discussed. Teachers who still believe such things too scandalous to be mentioned in the presence of the young, may find here just the temperate account they have been seeking. After all, college students are old enough not to be unduly shocked by the discovery that the Queen of the Sciences is no better than she should be.

So far, mathematics has appeared as a rather gentlemanly pursuit, interesting enough to those who like the sort of thing mathematics is (even if they do not know, and are not told, what it is), but essentially useless. Indeed, the emphasis throughout the book is on principles rather than on trite imitations of the genuinely practical applications of mathematics such as a modern engineer or theoretical physicist must know. In this connection, differentials (p. 435) as misused to great advantage by physicists, engineers and other practical men who get tangible things done, come in for some bad moments, and the hapless engineer who imagines his differentials as little bits of reality is left without a d to his x. Cannot something be done to span the chasm dividing unsound mathematics that helps to produce things that make money, from sound mathematics that barely pays a beggarly living wage to instructors in our colleges and universities? Rebuking the engineers and physicists for their differential and infinitesimal sins may relieve a pure mathematician's outraged emotions, but it will pay nobody's grocery bill, unless the mathematician is lucky enough to make a smash-hit with a best-selling text, as we hope this one will be.

Off the familiar highway are such things as items from topology in Chapters IV, V, where, incidentally, the Jordan curve theorem is described, perhaps to disabuse anyone who still takes too seriously the dictum of Gauss that "mathematics is the science of the eye." The balance between intuition and logical rigor, here as elsewhere, is justly preserved, and bigots for either intuition or logic as the sole guarantor of mathematical truth will find but little support for their exclusive dogmas. Interesting details that will be new to a majority

of the readers to whom the book is addressed are Brouwer's fixed point theorem, the genus of a surface, Klein's tantalizing bottle (the drunkard's deliverance), and the five color problem. To an algebraist it may seem somewhat antiquated to present only the analytic version of the fundamental theorem of algebra. Topology appears again in connection with minimax points; and generally the discussion of maxima and minima is as unhackneyed as anything in the book. Even Dirichlet's principle is described, possibly as a danger signal that intuition alone is not always enough, even in applied mathematics. Surely this must be an innovation in a college text. As the senior author has worked on Plateau's problem, he indulges his fancy for soap films in an interesting section suggesting that what mathematics slaves through pages of intricate analysis to accomplish, is sometimes done effortlessly by nature in a single gesture.

Enough has been sampled to indicate that here is an unusually rich text, full of suggestions for further reading, and replete with interesting and beautiful things not found in other books at the same level. The authors have reworked some of the best from each of many sources, due acknowledgement being made in the text and in the bibliography; the welding of it all into a homogeneous unity is exclusively their own. Sometimes the style is reminiscent of Klein; at other times Hilbert is plainly visible in the background; but all in all the work is an expression of independent individuality.

There is one aspect of the book which may be the most significant of all for young students looking forward to a career in mathematics. The emphasis of the recent past in American mathematics has been heavily on the side of axiomatics and abstraction; this book reminds us that intuition also has its part in mathematical creation.

Within the last decade several texts aiming to break away from the deadening drill in formal manipulation and theorems that are now only museum pieces have been written for American students. Some have favored mathematics as a logical discipline; others have presented it as a fine art; and yet others have described the power of mathematics in science and daily affairs. Each has had its definite function to perform, and all have contributed to a long overdue enlightenment in the teaching of mathematics at the college level. The present book differs in many respects from its predecessors; the points of difference make it a most welcome addition to the library of modernized teaching in mathematics.

E. T. Bell

Tables of the Moments of Inertia and Section Modulus of Ordinary Angles, Channels and Bulb Angles with Certain Plate Combinations. New York, Works Projects Administration, 1941. 13+197 pp. $1.25.

These tables are a result of a suggestion of the U. S. Bureau of Marine Inspection and Navigation, which is now a part of the Navy Department. The most important entries in these tables are the moments of inertia and the section moduli of rectangular areas with adjacent *L*-, Γ- or *C*-shaped areas (called, respectively, "angles" for the *L*- and Γ-shape and "channels" for the *C*-shape, the word "bulb angle" being reserved for an *L*-shaped area with a lateral circular swelling at the top). Both moments of inertia and section moduli are taken with respect to the neutral axis of the whole area parallel to the larger side of the rectangle. These entries were calculated by means of the parallel axis theorem, and the tables of the U. S. Steel Corporation were used for the moments of inertia and for the position of the centroid of the "angles" and "channels." The dimensions of the areas (or "sections" as they are technically called) which we find in these tables correspond to the commercial types, and—as one would expect—the tabular entries are not appropriate for difference checking. However, the accuracy of the values was insured through independent computing by two groups of workers, and was checked by graphing the entries. The calculations are carried out to two decimal places for small entries and are limited to integer digits for large entries (the units being $in.^4$ and $in.^3$). The arrangement of the tables is simple and handy, and the photo-offset reproduction gives them a very clear appearance. This publication will certainly be appreciated by many engineers.

I. OPATOWSKI

The Mathematical Papers of Sir William Rowan Hamilton. Vol. II. *Dynamics.* Edited for the Royal Irish Academy by A. W. Conway and A. J. McConnell. (Cunningham Memoir, no. 14.) Cambridge University Press; New York, Macmillan, 1940. 15+656 pp.

This volume contains Hamilton's work on dynamics and some investigations in optics. The period covered is, with two or three minor exceptions, that from 1833 to 1839. Except for a few short abstracts of papers read by Hamilton, most of the material presented has never before been published, and consists in the main, of a transcript of Hamilton's original work books and some scientific correspondence. The editors have added a few footnotes and some short explanatory material at the end of the volume.

The material is divided into three parts. A short introduction sketches the plan of the work and the nature of the problems investigated. The two hundred and eighty pages of Part I are devoted to dynamics and correspondence with Lubbock. Part II, consisting of some hundred pages, is devoted to the calculus of principal relations. Part III concerns itself with optical investigations and correspondence, and comprises some hundred and seventy pages. Also included in this volume are a thirty-two page appendix, contributed by the editors, and a very complete index.

The underlying idea in Part I is the determination of a single function whose partial derivatives will yield complete information about the trajectories. Hamilton uses at first the action function $\int 2T dt$ and applies it to the three body problem. In order to obviate the bothersome transformations that arise in considerations of the time, he perceives the advantage of using the principal function $S = \int L dt$ and indicates other alternative functions. It is in these investigations that the notion of varying action, the so-called canonical equations, and the Hamilton-Jacobi equation first come in as observations on the special dynamical situations treated, though, curiously enough, Hamilton makes no use of the canonical equations.

However, Hamilton requires that S satisfy a system of two partial differential equations, namely, the usual Hamilton-Jacobi equation and the corresponding equation where the differentiating variables are the initial coordinates and initial time. Now, at least when the force potential is independent of the time, the second equation is automatically satisfied whenever the first is, and Hamilton's lack of realization of this fact, quite obviously prevents him from making full application of the method. It is of course well known that it was only with Jacobi's recognition of this defect that the Hamiltonian method established its fundamental significance. (Apart from methodological novelty, Hamilton's dynamical and astronomical applications succeed only in verifying previously known computations.) Nevertheless in special cases Hamilton obtains an explicit principal function S and indicates an interesting method for this purpose. A complete integral $V(\cdots x_i \cdots ; \cdots a_i \cdots , t)$ of the Hamilton-Jacobi equation is a principal function if

$$\text{(1a)} \qquad V(\cdots \overset{0}{x_i} \cdots ; \cdots a_i \cdots ; t_0) = 0$$

and

$$\text{(1b)} \qquad L_{t \to t_0} \frac{V}{t - t_0} = L_{t \to t_0}\left(L\left(\cdots a_i \cdots , \cdots \frac{x_i - a_i}{t - t_0} \cdots ; t_0\right)\right)$$

$$\text{essentially } L_{t-t_0}\left(\frac{d}{dt}V - L\right) = 0.$$

Hence suppose

$$V = \sum V_i \tag{2}$$

where V is homogeneous of degree i in $x_i - a_i, t - t_0$. We may substitute (2) in the H.J. equation and expand $H(\cdots x_i \cdots, \partial V/\partial x_i \cdots, t)$ about x_i^0, t_0. On equating the linear and the quadratic terms, and so on, separately to zero, there results an infinite system of equations. The equation corresponding to terms of order n contains only V_j, $i \leqq j \leqq n$. On making use of (1) it is not difficult to show that the succeeding V_n's may be calculated step by step in terms of V_1. There are also some developments of perturbation and approximation methods for the determination of S.

The most ambitious appendix, quoted from a paper published by the editors, is devoted to the determination of the principal function from the knowledge of a complete integral of the H.J. equation. This is actually of slight importance, however, for it is well known that *any* complete integral determines the trajectories. Indeed (1a) suggests writing $S = V - V_0$, $V_0 = V(\cdots x_i^0 \cdots, \cdots a_i \cdots, t_0)$, and then the modification of the usual procedure replacing $b_i = \partial V/\partial a_i$ by $\partial V_0/\partial a_i = \partial V/\partial a_i$ for the elimination of the a_i's yields the principal function $S(\cdots x_i \cdots, \cdots x_i^0 \cdots, t, t_0)$.

Also in Part I is an ingenious derivation of the angular dependence of the resultant of two forces. The problem is reduced to the determination of an additive function f on the real numbers. Hamilton's argument to show $f(x)$ is linear and actually x overlooks the Hamel functions, but can readily be made rigorous.

The extensions of the dynamical methods, utilizing a principal function or relation, to arbitrary first and higher order partial differential and Mongean equations and systems, constitute the calculus of principal relations taken up in Part II. The following is typical. Consider

$$f_\sigma\left(x_1, \cdots \frac{d^{k_i}x^i}{dt^{k_i}}; \cdots; x_i \cdots \frac{d^{k_i}x_i}{dt^{k_i}}, \cdots; \cdots\right) = 0, \tag{3}$$

$$\sigma = 1, \cdots m;\ i = 1, \cdots n.$$

We associate with (3) the auxiliary equations of the calculus of variations. For instance with $\sigma = 1$, $k_i = k$ we have for a suitable choice of λ

$$(4)\qquad \lambda\frac{\partial f}{\partial x_i}-\frac{d}{dt}\lambda\frac{\partial f}{\partial \dot{x}_i}+\cdots+(-1)^k\frac{d^k}{dt^k}\lambda\frac{\partial f}{\partial x_i^k}=0,\quad x_i^k=\frac{d^k x_i}{dt^k}.$$

Consider now

$$(5)\qquad 0=\int_{t_0}^{t}\left(\delta f-\frac{df}{dt}\delta t\right)dt=I.$$

In view of (4) we may express the right side of (5) as a sum of terms of the form

$$(6)\qquad \left\{(-1)^\alpha\frac{d^\alpha}{dt^\alpha}\left(\lambda\frac{\partial f}{\partial x_i^{\beta+1}}\right)\frac{d^{\beta-\alpha}}{dt^{\beta-\alpha}}(\delta x_i-\dot{x}_i\delta t)\right\}\Bigg|_{t_0}^{t}.$$

If $F(\cdots;x_i\cdots d^{k-1}x_i/dt^{k-1};\cdots;\cdots;a_i\cdots a_i^{(k-1)};\cdots t,t_0)=0$ is the principal relation, then $\delta F=0$ is a consequence of (5) and hence

$$(7)\qquad \delta F=\mu I=0,$$

the central relation of this calculus. On equating the coefficients of the independent variations in (7) to 0 making use of (4), there arise auxiliary equations denoted by (8) for the partial derivatives of F with respect to $x_i^{(j)}$ and $x_i^{0,(j)}$ involving λ and μ also. When $k=1$ elimination of λ, μ, $\dot{x}_i$, $\dot{x}_i^0$ from these equations and (1) yields a differential equation for F and the equations (8) lead essentially to the bicharacteristics. For $k>1$ Hamilton indicates merely how the principal relation may be obtained. Thus to the equations (4) may be added a set of mn further equations obtained by replacing the left side of (4) by ith derivatives ($i=1,\cdots,n$) with respect to t. Then by successive eliminations we arrive at a single relation of the form specified above. Hamilton apparently did not attempt to determine the partial differential equation satisfied by F for $k>1$ and indeed the utility of the principal relation for higher order partial and Mongean equations is somewhat obscure. It would seem of interest to pursue these investigations.

The title assigned by the editors to Hamilton's treatment of higher order partial differential equations implies use of the principal relation but this is not clear from the text. He is led to relations obviously connected with the characteristic and bicharacteristic equations usually ascribed to Cauchy, Backlund, Goursat and Beudon and there seems to be some contact with Hamburger's investigations. However apart from the first order systems the results are incomplete and no account is taken of the elementary distinction between the elliptic, hyperbolic and parabolic cases.

The material on optics is perhaps of greatest interest. The propagation of disturbances may be taken up from two viewpoints. The first considers a compatible wave train already existing in the entire medium. Hamilton's interest is in the second viewpoint, namely that of the spreading of an originally localized disturbance. The typical problem is that of a collection of equi-spaced particles arranged over the entire x-axis and it is supposed that each particle acts only on its two adjacent neighbors.

The mathematical equivalent is the equation in mixed differences

$$y_{tt} - a^2 \frac{\Delta^2}{1+\Delta} y = 0, \qquad \Delta y(x, t) = y(x+1, t) - y(x, t)$$

subject to the initial conditions

$$y(x, 0) = 0 \text{ or } \eta(1 - \cos 2\nu x)$$
$$y_t(x, 0) = 0 \text{ or } -2a\eta \sin \nu \sin 2\nu x$$

for $x<0$ and $x\geqq 0$, respectively. If instead $y(x, 0)$ and $y_t(x, 0)$ vanish identically outside a finite interval, the conclusions are essentially unaffected. In treating these problems Hamilton makes incidental use of tools of modern appearance. For instance he uses the Fourier integral with the Sommerfeld weighting factor, the Abel summability of the Laplace integral, the Heaviside operator p^{-1}, and various operational manipulations, the Riemann-Lebesgue lemma, the method of stationary phase in conjunction with repeated use of the Dirichlet discontinuous integral (and incidentally obtains the leading term in the asymptotic expansion of the Bessel's function). If a, η are 1, then the nub of the analysis is the investigation of the properties of

$$\frac{1}{2\pi} (\sin \nu)^2 \int_0^\pi \frac{\sin (2x\theta - 2t \sin \theta)}{\sin \theta(\cos \theta - \cos \nu)} d\theta.$$

Assuming t large and positive, the results may be roughly summarized as follows. Let M and N be certain sufficiently large constants. Then for $x\geqq t+(1/2)Mt^{1/3}$, y is sensibly 0. As x decreases y takes on a pure displacement whose value is $(1/3)/(\cos \nu/2)^2$ for $x=t$ and rises to $(\cos \nu/2)^2$ for I: $t\cos \nu+(1/2)N(t\sin\nu)^{1/2}\leqq x\leqq t-(1/2)M_1t^{1/3}$. The displacement of amount $(1/3)(\cos \nu)^2$ travels with unit velocity. As x decreases still further an oscillatory disturbance sets in. Thus in the range II: $-t+(1/2)Mt^{1/3}\leqq x\leqq t\cos \nu-(1/2)N(t\sin \nu)^{1/2}$, $y=(1-\cos \phi-(\sin \nu/2)^2$, $\phi=2\nu x-2t \sin \nu$. This is a copy of the initial disturbance shifted to a new mean position. In the transition region linking I and II the disturbance is a shifted, uniformly attenuated

copy of the original. With further decrease in x, the effect is merely to change the mean position of the vibrations and for III: $x \leqq -t-(1/2)Mt^{1/3}$, $y=\cos\phi$. Hamilton adopts the view that it is the region of oscillatory disturbance that is significant. He chooses the value $x=t\cos\nu$ in the transition region T between I and II for the wave front. Here $y=(1/2)(1-\cos\phi+\cos\nu)$ and $\cos\nu$ is a dispersive (group) velocity. This is not altogether capricious, for with increasing t the velocity throughout T approaches $\cos\nu$, and the ratio of the size of T to the time approaches zero.

It has not been realized that the modern work on the transient phenomena in the propagation of waves in continuous media by Sommerfeld, Brillouin, and Colby is anticipated in these researches. (Even the reference to the published abstract of Hamilton's work, in Havelock's well known Cambridge tract, overlooks this fact.) There is moreover a marked parallelism in the methods, the stationary phase calculations being superseded by the more delicate saddle point evaluations. Actually, the adoption of a discrete rather than a continuous medium has certain advantages in indicating more intuitively the nature and genesis of the conclusions. Hamilton extends his work to two-dimensional and three-dimensional problems using the now familiar Cauchy-Fourier integral methods, and gives the explicit general formula for the group velocity.

The editors have obviously presented a most felicitous and careful rendering of Hamilton's work. However, it would seem desirable in collections of hitherto unpublished material to give a succinct summary of the *details* of (a) methods and (b) results in modern terminology and, wherever possible, (c) some indication of the present status of the field covered. The editors have made an attempt in this direction for (a) and (b), particularly as regards the dynamical papers, but the appendices and footnotes, could, with profit, be expanded considerably. The viewpoint and incomplete character of many of the investigations may well attract the attention of mathematicians ordinarily unconcerned with the historical development of mathematics.

D. G. BOURGIN

NOTES

The organization called VOKS (or Society for Cultural Relations with Foreign Countries, B Gruzinskaya 17, Moscow 56, U.S.S.R.) is able to locate institutions evacuated because of the war and has as its purpose the continuation of scientific contact between Russia and other countries. The Embassy advises us to send letters to individual scientists through VOKS.

Professor R. H. Fowler of the University of Cambridge and Dr. R. P. Paranjpye of Poona, India, have been knighted.

Dean L. P. Eisenhart of Princeton University has been appointed Executive Officer of the American Philosophical Society to succeed Professor Emeritus E. G. Conklin of Princeton University, who is now President of the Society.

Professor H. B. Evans of the University of Pennsylvania has retired with the title professor emeritus.

Professor A. C. Lunn of the University of Chicago has retired with the title professor emeritus.

Professor C. B. Upton of Teachers College, Columbia University, has retired with the title professor emeritus.

Dr. J. S. de Wet of the University of Pretoria has been promoted to a professorship.

The following members of the American Mathematical Society are in residence at the Institute for Advanced Study for all or part of this academic year: Dr. Valentine Bargmann, Dr. I. S. Cohen, Dr. Kurt Gödel, Dr. Audrey W. McMillan, Dr. W. R. Transue, Dr. L. I. Wade and Mr. J. E. Wilkins.

The following persons are also to be at the Institute for Advanced Study for all or part of this academic year: Dr. Shuichi Kusaka, Professor Wolfgang Pauli, formerly of the Technical School in Zurich, and Professor C. L. Siegel, formerly of the University of Göttingen.

Assistant Professor H. W. Bailey of the University of Illinois has been promoted to an associate professorship.

Professor B. L. Beegle of Seattle Pacific College has been appointed dean of the college.

Assistant Professor P. G. Bergmann of Black Mountain College, Black Mountain, North Carolina, has been appointed to an assistant professorship of physics at Lehigh University.

Dr. Elizabeth M. Boggs of Cornell University has been appointed to a lectureship in physics at the University of Pittsburgh.

Dr. H. E. Burns of Indiana University Extension, East Chicago, Indiana, has been promoted to an assistant professorship.

Associate Professor H. V. Craig of the University of Texas has been promoted to a professorship.

Assistant Professor M. W. Dehn of the University of Idaho has been appointed visiting lecturer at the Illinois Institute of Technology.

Dr. H. C. Dieckmann of Occidental College has been promoted to an assistant professorship.

Professor J. L. Dorroh of Ouachita College, Arkadelphia, Arkansas, has been appointed to a visiting assistant professorship at Louisiana State University.

Associate Professor H. P. Evans of the University of Wisconsin has been promoted to a professorship.

Assistant Professor Willy Feller of Brown University has been promoted to an associate professorship.

Associate Professor I. H. Fenn of the Polytechnic Institute of Brooklyn has been promoted to a professorship.

Dr. A. D. Fialkow of Brooklyn College has been appointed to a lectureship in the School of Pharmacy at Columbia University.

Dr. F. A. Ficken of Cornell University has been appointed to a professorship at the University of Tennessee.

Assistant Professor H. A. Giddings of the Illinois Institute of Technology has been appointed to an associate professorship at the University of New Hampshire.

Dr. G. L. Gross of the Agricultural and Mechanical College of Texas has been promoted to an assistant professorship.

Dr. Margaret Gurney of Wykeham Rise School, Washington, Connecticut, has been appointed associate economist in the Bureau of the Budget, Washington.

Mr. M. G. Harrison has accepted a position as consultant at Republic Aviation Corporation.

Dr. O. G. Harrold of Northwestern University has been appointed to an assistant professorship at Louisiana State University.

Assistant Professor Frances Harshbarger of Kent State University has been promoted to an associate professorship.

Dr. G. E. Hay of the University of Michigan has been promoted to an assistant professorship.

Associate Professor E. A. Hedberg of Baylor University is on leave at the United States Naval Pre-Flight School in Athens, Georgia.

Assistant Professor M. R. Hestenes of the University of Chicago has been promoted to an associate professorship.

Dr. Aughtum S. Howard of the University of Kentucky has been appointed to an associate professorship at Kentucky Wesleyan College.

Mr. J. D. Hutchinson of the University of Houston has been promoted to an associate professorship.

Dr. William Karush of the University of Chicago has accepted a position at Carnegie Institution of Washington.

Dr. M. W. Keller of Purdue University has been promoted to an assistant professorship.

Dr. J. C. Knipp of the University of Pittsburgh has been promoted to an assistant professorship.

Dr. C. F. Kossack of the University of Oregon has been promoted to an assistant professorship.

Associate Professor V. S. Lawrence of Cornell University has been commissioned major in the United States Army and will teach mathematics at the United States Military Academy at West Point.

Dr. Charles Loewner of the University of Louisville has been promoted to an assistant professorship.

Dr. Eugene Lukacs has been appointed to an assistant professorship at Illinois College.

Dr. Ingo Maddaus of the University of Oklahoma has been appointed to an assistant professorship at the University of Oregon.

Dr. P. T. Maker of Rutgers University has been appointed to an assistant professorship at Duke University.

Dr. Herman Meyer of the Illinois Institute of Technology has been appointed to an assistant professorship at the University of Miami.

Dr. Harlan C. Miller of Winthrop College has been appointed to an associate professorship at North Texas State Teachers College.

Mr. B. C. Moore of the Agricultural and Mechanical College of Texas has been promoted to an assistant professorship.

Associate Professor Sara L. Nelson of Georgia State College for Women has been promoted to a professorship.

Dr. E. N. Nilson of the University of Maryland has been appointed to an assistant professorship at Mount Holyoke College.

Reverend R. E. O'Connor has been appointed to a professorship at Loyola College, Montreal, Quebec.

Dr. H. L. Olson of Itasca Junior College has been appointed to an assistant professorship at Southwestern University, Georgetown, Texas.

Dr. W. T. Puckett of the University of California at Los Angeles has been promoted to an assistant professorship.

Assistant Professor W. T. Reid of the University of Chicago has been promoted to an associate professorship.

Dr. Haim Reingold of Our Lady of Cincinnati College is on leave at the Illinois Institute of Technology.

Professor W. J. Rusk of Grinnell College has retired.

Professor Emeritus Virgil Snyder of Cornell University has been appointed to a visiting professorship at Brown University.

Dr. Domina E. Spencer has been appointed to an assistant professorship at the American University.

Assistant Professor D. V. Steed of the University of Southern California has been promoted to an associate professorship.

Dr. Alfred Tarski has been appointed lecturer at the University of California in Berkeley.

Dr. R. M. Thrall of the University of Michigan has been promoted to an assistant professorship.

Dr. F. A. Valentine of the University of California at Los Angeles has been promoted to an assistant professorship.

Dr. R. K. Wakerling of Texas Technical College has been appointed to an assistant professorship at Fresno State College, Fresno, California.

Professor Evelyn P. Wiggin of Randolph Macon Woman's College is on leave as lecturer at Wellesley College.

The following appointments to instructorships are announced: Angleton Public Schools, Angleton, Texas: Mr. S. K. Bright; Bowdoin College: Dr. D. E. Christie; University of California: Dr. Dorothy L. Bernstein; Case School of Applied Science: Dr. L. J. Green; University of Cincinnati: Mr. Joshua Barlaz; Cornell University: Dr. Fred Kiokemeister, Dr. L. J. Savage; University of Florida: Dr. G. B. Lang; Hamilton College: Mr. L. B. Williams; Houghton College: Dr. R. R. R. Luckey; John Carroll University: Dr. P. H. Anderson; University of Kansas: Dr. G. K. Kalisch; Massachusetts Institute of Technology: Dr. P. R. Wallace; Michigan State College: Dr. D. E. Kibbey, Dr. W. L. Mitchell; U. S. Naval Air Station, Pensacola, Florida: Mr. C. C. Dearman; U. S. Naval Pre-Flight School, Iowa City, Iowa: Dr. Howard Levi; Northwestern University: Dr. Zenon Szatrowski; University of Oklahoma: Mr. W. J. Dixon; Pennsylvania State College: Mr. E. H. Umberger; Princeton University: Dr. Brockway McMillan, Dr. E. W. Montroll; Purdue University: Dr. Leonidas Alaoglu, Dr. J. H. Giese, Dr. Michael Golomb, Dr. G. S. Young; Randolph Field, Texas: Dr. C. L. Seebeck; St. Olaf College: Mr. W. S. Erickson; U. S. Military Academy: Mr. B. M. Ingersoll; University of Wisconsin: Mr. P. B. Norman; Yale University: Dr. Edward Begle, Mr. A. W. Jones.

Professor Emeritus A. R. Forsyth of the Imperial College of Science and Technology, South Kensington, died on June 2, 1942, at the age of eighty-three years.

The death of Professor W. H. Young, formerly professor of mathematics at the University of Wales, has been announced.

Professor Emeritus Eleanor C. Doak of Mt. Holyoke College died August 27, 1942, at the age of seventy-two years. She became a member of the Society in 1916.

Professor Emeritus W. A. Harshbarger of Washburn University died on July 17, 1942, at the age of seventy-nine years. He had been a member of the Society since 1909.

Dean T. M. Putnam of the University of California died September 22, 1942. He had been a member of the Society since 1902 and had served as Associate Secretary in charge of meetings on the Pacific Coast since 1930.

Professor J. F. Reilly of the State University of Iowa died August 18, 1942. He had been a member of the Society since 1914.

Dr. W. M. Strong, who retired about a year ago as vice president and actuary of Mutual Life Insurance Company of New York, died March 30, 1942, at the age of seventy-one years.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA AND THEORY OF NUMBERS

299. Leonard Carlitz: *Representation of a polynomial in certain forms.*

Explicit formulas are found for the representation of an irreducible polynomial P in $GF(p^n)$ in certain factorable forms of degree k in k indeterminates. For example, when $k=2$, $p\neq 2$, the form may be taken as $U^2-\alpha V^2$, where α is not a square in $GF(p^n)$. Here U is determined by the formula $2A(x)\equiv(-1)^h[1][3]\cdots[2h-1] \pmod{P(x)}$, where P is of degree $2h$, and $[i]=x^{p^{ni}}-x$ (Duke Mathematical Journal, vol. 9 (1942), pp. 234–243, p. 242). Similar results are obtained for the general case. (Received August 3, 1942.)

300. Leonard Carlitz: *Some formulas for the composition of numerical functions.*

This note is concerned with the sum $h(M)=\sum f(U)g(V)$ extended over polynomials of fixed degree k in $GF(p^n)$ such that $(\alpha+\beta)M=\alpha U+\beta V$, α, β in $GF(p^n)$; it is assumed that $f(U)$ and $g(V)$ are of the form $\sum e(D)$, where D runs through the divisors of U of degree at most $k/2$. It is found that $h(M)$ is of the same general form. In various special cases $h(M)$ can be expressed in quite simple form. In particular the formulas of the present paper include formulas occurring in the problem of representing a polynomial as the sum of an even number of squares. (Received August 3, 1942.)

301. R. P. Dilworth: *On the decomposition theory of modular lattices.*

The Kurosch-Ore decomposition theorem asserts that if an element a of a modular lattice has two reduced decompositions into irreducibles $a=p_1\cap\cdots\cap p_m=q_1\cap\cdots\cap q_n$, then each p_i may be replaced by a suitably chosen q_j. However, this leaves unanswered questions like the following: Does each q_j replace some p_i? Can different p's be replaced by different q's? The following precise result is proved: If $a=p_1\cap\cdots\cap p_m=q_1\cap\cdots\cap q_n$, then the q's can be renumbered in such a way that each p_i may be replaced by q_i. This theorem requires a much deeper analysis of the arithmetical structure of a modular lattice. For this purpose extensive use is made of the concept of an *over-divisor* which is defined as follows: a is said to *over-divide* b, or to be an *over-divisor* of b if $x\cap a=b$ implies $x=b$. (Received August 4, 1942.)

302. C. J. Everett: *Sequence completion of lattice moduls.*

A lattice modul L (G. Birkhoff, *Lattice ordered groups*, Annals of Mathematics, (2), vol. 43) is linear if and only if $a>0$, $b>0$ implies $a\wedge b>0$; hence the neighborhood topology $N_e(a)=(x;\ |x-a|<e,\ e>0)$ defines a topological group satisfying the crucial Hausdorff space intersection axiom if and only if L is linear. o-convergence and o-regular sequences are defined, and necessary and sufficient conditions given for every o-regular sequence to o-converge. Such an L is called o-complete. The first fundamental extension L' of L by classes of regular sequences imbeds L with preservation of $+$, unrestricted $\vee$, $\wedge$, and o-convergence. If L admits diagonalization (G. Birkhoff, *Lattice Theory*, American Mathematical Society Colloquium Publications, vol. 25, 1940, p. 28), then L' is o-complete. The MacNeille completion $\mathfrak{C}$ of the lattice L by cuts contains a maximal lattice submodul $\mathfrak{M}\supseteq L$ of all elements of $\mathfrak{C}$ having inverses. $\mathfrak{M}$ is o-complete and imbeds L with the same preservations as for L'. L' is isomorphic to the set L_1 of the elements of $\mathfrak{M}$ which are o-limits of L-sequences relative to L similarly L'', L''', and so on. A sequence is o-regular in L if and only if it o-converges in $\mathfrak{M}$ relative to L. If L is linear, o-convergence is equivalent to sequence convergence relative to $N_e(a)$ topology; L_1 is o-complete, indeed $L_1=\mathfrak{M}$ whenever there exist non-zero null sequences in L. (Received October 1, 1942.)

303. Irving Kaplansky: *Systems of congruences in a valuation ring.*

Infinite systems of linear congruences, introduced by Krull in his general definition of completeness of fields with valuations, are here used to obtain a characterization of maximality. As an application, a classical result is generalized; under the hypothesis that the field K is maximal in a valuation, it is shown that the degree of any extension of K is equal to the product of the ramification order and residue class degree. (Received September 29, 1942.)

304. Irving Kaplansky: *The direct product of rings.*

Whitney's tensor product $G\circ H$ of abelian groups (Duke Mathematical Journal, vol. 4 (1938), pp. 495–528), reduced with respect to a ring of operators R, provides a natural definition of the direct product of rings. If G and H are imbedded in a larger ring, the problem arises of deducing the usual criterion that the product GH be direct; this can be done if certain restrictions are made, as indicated by Dorroh in the case where R is the ring of integers (Annals of Mathematics, (2), vol. 36 (1935), pp. 882–885). This general notion of direct product is used to obtain a variety of results, some generalizations of known theorems, others new. (Received September 29, 1942.)

305. A. J. Kempner: *Periodic decimals to any base, and quadratic residues.*

Despite the large literature on the periods of decimal fractions there does not seem to exist any unified theory of this very special field. Restriction to the base 10 or to any individual base q is one reason for this deficiency. In order to gain a broader basis it is necessary to consider the representation of *all* proper fractions for *all* bases. In this enlarged formulation of the problem there is room for a satisfying theory. To bring out the structure of the system it is of advantage to place emphasis on the system of remainders $a_1, \cdots, a_\mu$ rather than on the system of digits $\alpha_1, \cdots, \alpha_\mu$ in the algorithm for the expansion of a/b to base q ($a\cdot q=\alpha_1\cdot b+a_1$, $a_1\cdot q=\alpha_2\cdot a_1+a_2$, $\cdots$, $a_{\lambda-1}\cdot q=\alpha_\lambda\cdot a_{\lambda-1}+a_\lambda$, $\cdots$). The a's possess a period of the same length μ as do the α's,

but have the advantage of being all distinct in a period, thus leading to a more incisive classification than is possible with the digits. (Received August 4, 1942.)

306. Gordon Pall: *The distribution of integers represented by binary quadratic forms.*

The formula due to R. D. James (American Journal of Mathematics, vol. 60 (1938), pp. 737–744) for the number of integers m prime to d represented by binary quadratic forms of discriminant d, is here freed of the restriction that m be prime to d. (Received August 3, 1942.)

307. Gordon Pall: *The weight of an n-ary genus of quadratic forms.*

The formula for the weight of a genus of integral positive quadratic forms in n variables is obtained in an explicit, useful form. The calculation of the factor for $p=2$ is greatly facilitated by the use of a much simplified system of invariants. There are numerous applications. (Received August 3, 1942.)

308. J. F. Ritt: *Bezout's theorem and algebraic differential equations.*

The intersection of the general solutions of two differential polynomials in two unknowns is examined with respect to the numbers of arbitrary constants on which its various irreducible components can depend. (Received August 6, 1942.)

309. Ernst Snapper: *The resultant of a linear set.*

The m-dimensional vector space $\overline{V}_m$ consists of vectors having, as components, m polynomials of the ring $P[y_1 \cdots y_n]$ where P is a field. The linear subsets of $\overline{V}_m$ are generated by the columns of $m \times s$ matrices with elements in $P[y_1 \cdots y_n]$. The ideal theory of $P[y_1 \cdots y_n]$, given by Hentzelt and Noether (Mathematische Annalen vol. 88 (1922), pp. 53–79), holds for these linear sets. By a linear, invertable transformation of the variables $y_1, \cdots, y_n$, which involves adjoining new variables γ_{ij} to P, the linear subsets of $\overline{V}_m$ become "transformed" linear sets of the vector space V_m over $P(\gamma)[x_1 \cdots x_n]$. Every transformed linear set L of V_m has a resultant $\rho \in P(\gamma)[x_1 \cdots x_n]$, which vanishes for, and only for, the zeros of the ideal $L/\bar{L}$. (See Snapper, Transactions of this Society, vol. 52 (1942), pp. 258–259 for the definitions of $\bar{L}$ and $L/\bar{L}$.) If $L_1 \subseteq L_2$, then $L_1 = L_2$, if and only if they have equal ranks and resultants. This gives a criterion for the existence of a polynomial solution of simultaneous linear equations with polynomials as coefficients. For $n=1$, the resultant becomes the highest dimensional determinantal factor of L. (Received September 29, 1942.)

Analysis

310. M. A. Basoco: *On the Fourier developments of a certain class of theta quotients.*

This paper is concerned with the functions $\phi_\alpha^k(z) \equiv \{\vartheta_\alpha'(z, q)/\vartheta(_\alpha z, q)\}^k$ $(\alpha = 0, 1, 2, 3)$ where $\vartheta_\alpha(z, q)$ is a Jacobi theta function and k is a positive integer. The Fourier expansions of these functions are investigated and their arithmetical form is obtained for the cases $k=1, 2, 3$. Using these results and certain simple identities, there is obtained using the method of paraphrase (E. T. Bell, Transactions of this Society, vol. 22 (1921), pp. 1–30 and 198–219; *Algebraic Arithmetic*, American Mathematical Society Colloquium Publications, vol. 7, 1927, chap. 3), a series of theorems on numer-

ical functions. Among these there appears a theorem due to Liouville (Journal de Mathématiques Pures et Appliquées, (2), vol. 3 (1858), p. 247), which has only recently been proved by elementary means by J. V. Uspensky (*Elementary Number Theory*, New York, 1939, p. 462). An application of these theorems leading to relationships between infinite series of the Lambert type is indicated. This paper will appear shortly in this Bulletin. (Received September 26, 1942.)

311. L. W. Cohen: *Integration on hypersurfaces.*

An invariant measure is developed for m-dimensional surfaces Σ_m which yields proofs of the theorems of Gauss and Stokes in terms of Lebesgue integrals. A surface Σ_m is a complex of m-cells $R_x^m = X(R_u^m)$ where R_u^m is an m-dimensional interval and $X = X(U)$ is a 1-1 mapping of R_u^m on an arbitrary set. To each $X \subset R_x^m$ are assigned numbers $x_1, \cdots, x_m$, $n \geqq m$, which are almost everywhere differentiable functions of the coordinates of $U \subset R_u^n$. The measurable sets $E_x \subset R_x^m$ are the images of the measurable sets $E_u \subset R_u^n$. The measure of E_x is the integral of $D_m^{[n]}(U)$ over E_u where $(D_m^n(U))^2$ is the Gram function of the $\partial x_i/\partial u_j$. The measure on Σ_m is determined by the measure on its several cells. For the Gauss theorem, Σ_m need not be orientable and may have singularities on its boundary while the functions are summable on the boundary and have finite summable derivatives on Σ_m. The Stokes theorem states the invariance of a certain integral on a family of homologous surfaces. (Received August 12, 1942.)

312. Willy Feller: *On some geometric inequalities.*

The following problem was originally formulated by R. Salem and D. C. Spencer in connection with a number-theoretical investigation. Consider a domain Γ contained in the unit sphere of R_n; suppose that the intersection of Γ with any straight line has a measure not exceeding a fixed constant $\delta < 1$. The problem is to determine the maximum measure of Γ. In the present paper this and a more general analytical problem are solved. At the same time an extremely simple proof is given for the well known inequality for the measure of convex sets with the above property (due to Bieberbach and Kubota). (Received August 3, 1942.)

313. Einar Hille and Gabor Szegö: *On the complex zeros of the Bessel functions.*

This paper contains a new proof of the theorem of A. Hurwitz according to which $z^{\beta/2}J_{-\beta}(2z^{1/2})$ has exactly $[\beta]$ non-positive zeros when $\beta \geqq 0$. The proof is based upon the fact that $n^{\beta}L_n^{(-\beta)}(z/n)$ tends to the function in question and the number and position of the non-positive zeros of Laguerre polynomials of negative order are easily discussed. (Received August 3, 1942.)

314. Abraham Hillman and H. E. Salzer: *Complex roots of* $\sin z = z$.

The first ten nonzero roots of $\sin z = z$ in the first quadrant were computed to seven decimal places. Obviously the roots are symmetrically situated in the four quadrants. The imaginary part, y, of the nth root satisfies the equation $x_1(y) - x_2(y) = 0$, where $x_1 = \coth y(\sinh^2 y - y^2)^{1/2}$ and $x_2 = 2n\pi + \arccos(y/\sinh y)$. The y of the root was found by calculating $x_1 - x_2$ in the neighborhood of G. H. Hardy's approximation $y = \log(4n+1)\pi$ and then using inverse interpolation. The real part, x, was then taken as $x_2(y)$. The x and y were checked by the relation $x = x_1(y)$. (Received September 30, 1942.)

315. Ervand Kogbetliantz: *Inequality for definite integrals.* Preliminary report.

It is shown that the inequality for definite integrals $\int_a^b f(x)g(x)dx$ recently established by Shohat admits of a simple geometrical interpretation. This permits the extension of the inequality to the case where first n moments of $f(x)$ and $g(x)$ in $[a, b]$ vanish. (Received September 30, 1942.)

316. D. M. Krabill: *On extension of Wronskian matrices.*

The Wronskian matrix of order $n-1$ for a given set of solutions of a differential equation of the type (H): $y^{(n)}+f_1(x)y^{(n-1)}+\cdots+f_n(x)y=0$ with the $f_i(x)$ real and continuous in an interval J, has constant rank r in J and every column is a linear combination with constant coefficients of some r of the columns. The author gives a sufficient condition that an arbitrary set of functions having suitable class properties be solutions of an equation of type (H). The principal theorem states: If $k \leqq s \leqq t$ and the Wronskian matrix is of order s, $M_s(u_1, \cdots, u_k)$, for k arbitrary functions of class $C^{(t)}$ in J has constant rank k, there exists a function u_{k+1} of class $C^{(t)}$ with the extended matrix $M_s(u_1, \cdots, u_{k+1})$ of rank $k+1$ throughout J. A theorem of Curtiss (Mathematische Annalen, vol. 65 (1908), pp. 282–298, Theorem 4) on the zeros of the Wronskian determinant $W(u_1, \cdots, u_k)$ is utilized. From the extension property of Wronskian matrices is obtained the result: If the Wronskian matrix of order $n-1$ for k functions of class $C^{(t)}$ $(k \leqq n \leqq t)$ in an interval J has constant rank k, then the u's are linearly independent solutions of an equation of type (H) with the $f_i(x)$ of class $C^{(t-n)}$ in J. (Received August 3, 1942.)

317. Lee Lorch: *Lebesgue constants for Borel summability of Fourier series.*

Let $L_B(u)$ denote the uth Lebesgue constant for Borel summability of Fourier series. C. N. Moore (Proceedings of the National Academy of Sciences, vol. 11 (1925)) remarked that $L_B(u)$ becomes logarithmically infinite with u. Here it is shown that $L_B(u) = (2/\pi^2)\log u - (2/\pi^2)\log(\pi^2/2) - 2C/\pi^2 - (2/\pi^2)\int_0^\infty \psi(t/\pi)\sin t\,dt + O(u^{-1/2})$ as u becomes infinite, where $\psi(t) = \Gamma'(t)/\Gamma(t)$ and C is the Euler-Mascheroni constant. (Received August 3, 1942.)

318. Szolem Mandelbrojt: *Quasi-analyticity and properties of flatness of entire functions.*

The author proves a general theorem which may be regarded as a theorem concerning quasi-analyticity as well as the behaviour of entire functions on a set of points. This theorem relates an integrable function to an entire function. Each time that an entire function is chosen there results a theorem on quasi-analyticity, and when a suitable integrable function is given a theorem concerning entire functions follows. This paper will appear in the Duke Mathematical Journal. (Received September 14, 1942.)

319. W. T. Reid: *Some results on the growth of solutions of differential systems.*

In this paper elementary differential and integral inequalities are used to prove various results on the growth of solutions of differential systems, both linear and nonlinear, and involving functions of a real independent variable. In particular, the

methods herein employed afford very brief proofs and improved forms of many of the results recently established by Trjitzinsky (Transactions of this Society, vol. 50 (1941), pp. 252–294). The paper also contains applications of an improved form of a theorem on matrix transformations due to Perron (Mathematische Zeitschrift, vol. 32 (1930), pp. 465–473). (Received September 7, 1942.)

320. R. M. Robinson: *Bounded analytic functions.*

(1) Suppose $g(z)$ is regular and satisfies $|g(z)| \leqq 1$ for $|z| < 1$. The relations between $g(0)$, z_0, $g(z_0)$, and $g'(z_0)$ are studied. For example, it is shown that for given values of $g(0)$ and z_0, the possible values of $g'(z_0)$ fill a closed convex set, which is bounded by a circle, a limaçon, or partly by one and partly by the other. The relations between the absolute values of the four quantities mentioned are also considered. (2) Suppose $f(z)$ satisfies the same conditions as $g(z)$, and also $f(0)=0$. The relations between $f'(0)$, z_0, $f(z_0)$, $f'(z_0)$ are studied. The relations between their absolute values are also considered. For example, the sharp lower and upper bounds for $|f'(z_0)|$ in terms of $|f'(0)|$ and $|z_0|$ are found. The lower bound is given by $|f'(z_0)| \geqq [|f'(0)| \cdot (1+|z_0|^2)-2|z_0|]/(1-|f'(0)z_0|)^2$ provided the right side is positive, and is 0 otherwise; the upper bound is given by four formulas, valid for different values of $|f'(0)|$ and $|z_0|$. The relations between $|f'(0)|$, $|z_0|$, $|f(z_0)|$, and $|f'(z_0)|$, for univalent functions, were studied by the author in an earlier paper (Transactions of this Society, vol. 52 (1942), pp. 426–449). (Received September 16, 1942.)

321. P. C. Rosenbloom: *The zeros of the partial sums of power series representing entire functions.*

Let $f(z)$ be an entire function of order $0 < \rho \leqq \infty$. Consider the zeros of the nth partial sum $s_n(z)$ for large n. If $\rho = \infty$, it is proved that there are a sequence $\{n_i\}$ and a sequence of radii $\rho_i \to \infty$ such that if ϵ, $\delta > 0$, then there is an $i_0 = i_0(\epsilon, \delta, f)$ for which the number of zeros of $s_{n_i}(z)$ in the ring $\rho_{n_i}(1-\delta) \leqq |z| \leqq \rho_{n_i}(1+\delta)$ is $n_i(1-\eta)$ with $0 \leqq \eta < \epsilon$. The arguments of the zeros of $s_{n_i}(z)$ are equally distributed in the interval $0 \leqq \theta \leqq 2\pi$. If R_{n_i} is the maximum modulus of the zeros of $s_{n_i}(z)$, then $\limsup R_{n_i}/\rho_{n_i} \leqq 2$. The radius ρ_{n_i} can be chosen as $|a_{n_i}|^{-1/n_i}$ where a_{n_i} is the nth coefficient in the Taylor series for $f(z)$. These results constitute a sharpening of some theorems of Carlson (Comptes Rendus de l'Académie des Sciences, Paris, 1924), whose proofs have never been published (so far as we know). These results bear a striking analogy to the results of Jentzsch (*Untersuchungen zur Theorie der Folgen analytischen Funktionen*, Dissertation, Berlin, 1914) and Szegö (*Über die Nullstellen der Polynomen, die in einem Kreise gleichmässig konvergieren*, Sitzungsberichte der Berliner Mathematische Gesellschaft, vol. 21 (1922)) for power series with a finite radius of convergence. For functions of finite order, $\limsup R_{n_i}/\rho_{n_i} \leqq 2 \exp(1/\rho)$ for a certain sequence $\{n_i\}$. The paper also includes a detailed investigation of certain functions of finite order such as the error function, with results similar to Szegö's, for the exponential function (*Über eine Eigenschaft der Exponentialreihe*, Sitzungsberichte der Berliner Mathematische Gesellschaft, vol. 23 (1924)). (Received August 31, 1942.)

322. Max Shiffman: *Unstable extremal surfaces for double integral problems in the calculus of variations.*

Theories of the absolute minimum for double integral problems have been established in recent years. In this paper the author develops a theory of unstable extremal

surfaces, proving the general Morse relations. The integrals to be made stationary have the form $\iint\{f(X, Y, Z)+k(X^2+Y^2+Z^2)^{1/2}\}\,dudv$ where X, Y, Z are the Jacobians $(y_u, z_u; y_v, z_v)$, and so forth, k is a positive constant, and f satisfies the following four conditions: (1) f is positively homogeneous in X, Y, Z; (2) $\epsilon=f(X, Y, Z) - \{Xf_{x_0}+Yf_{Y_0}+Zf_{z_0}\}\geqq 0$; (3) $f(X, Y, Z)\geqq m>0$ for $X^2+Y^2+Z^2=1$; and (4) $f(X, Y, Z)<k$ for $X^2+Y^2+Z^2=1$. The first three are usual conditions to be expected; the main restriction is (4) which asserts that the integral is dominantly an area integral. The problem is to find extremal surfaces bounded by a given rectifiable curve. The integral is replaced by $\iint f(X, Y, Z)dudv+kD[x]$ where $D[x]$ is the Dirichlet integral, and the extremal surfaces found are given in isometric representation. (Received August 5, 1942.)

323. J. A. Shohat: *An inequality for definite integrals.* Preliminary report.

Starting with a simple case of the Schwarz inequality, another inequality is derived in a very elementary manner. This inequality seems to be new and may be considered an improvement over the Schwarz inequality. In fact, it is shown that the new inequality yields better results than the Schwarz inequality (it reduces to the latter, in some special cases).—Applications are indicated to orthogonal functions and to polynomials. (Received September 17, 1942.)

Applied Mathematics

324. Lipman Bers and Abe Gelbart: *On solutions of the differential equations of gas dynamics.* I.

By a transformation due to Chaplygin and Busemann the equations of the steady two-dimensional irrotational flow of a gas can be reduced, in the subsonic case, to the form (1): $u_x=\tau(y)v_y$, $u_y=-\tau(y)v_x$, where τ is a given function. Equations of the same form occur in other branches of mechanics of continua. The class of complex-valued functions $f(x+iy)=u+iv$, where u and v satisfy (1) is shown to have many properties in common with analytic functions. Two processes, one the inverse of the other, are introduced: τ-differentiation and τ-integration. The τ-derivative and the τ-integral of f belong to the class. By τ-integrating a complex constant, a_n, n times a function $a_n\cdot Z\cdot{}^n(z)$, of the class is obtained. From these "powers," " polynomials" and "power series" are formed. A "polynomial" of the nth degree possesses n zeros. Any function, f, of the class can be developed in a unique manner in a power series, the "coefficients" a_n, being given by the nth τ-derivatives of f. Domains of convergence, mapping properties, and particular functions are considered. Analogous methods are applied to more general types of equations; in particular to the system $u_x=\tau_1(y)v_y$, $u_y=-\tau_2(y)v_x$ which describes the subsonic as well as supersonic flow. (Received August 27, 1942.)

325. Garrett Birkhoff: *A reversibility paradox in hydrodynamics.*

It is shown that every known differential equation and general boundary condition describing compressible non-viscous flow is reversible, in the sense that it is compatible with reversal of the direction of flow without change in pressure. This is even true in the presence of a conservative force-field. It follows that, contrary to a widespread impression, any theory of non-viscous flow (airfoil theory, resistance to projectiles, or resistance to surface craft in water) which purports to be based entirely on

general considerations, must predict that a flow and its reverse give identical pressure distributions, hence identical lift and drag. This reversibility paradox partly overlaps the d'Alembert paradox, but it is far simpler, and more general in that it applies to modern airfoil theory and to ships. Special methods for avoiding it under particular circumstances, and their comparative validity, are then discussed. (Received August 27, 1942.)

326. Nathaniel Coburn: *The linear yield condition in the plane plasticity problem.*

The curvilinear nets consisting of curves of maximum and minimum shearing stress have been studied, extensively. However, all of these studies have assumed that the material under investigation follows the von Mises yield condition. In this paper, the above nets are investigated for materials which follow a linear yield condition. It is shown that two cases exist: (1) corresponding to lines of slope ± 1; (2) corresponding to lines of slope other than 0, ± 1 (the case of slope 0 is that due to von Mises). For the first case, it is shown that the two families of curves of the net possess the following two properties: (1) they constitute an equiareal system; (2) at any given point, the magnitudes of their radii of curvature are equal. Analytically, the functional relation is determined which the metric coefficients satisfy. For the second case, it is shown that a generalization of the von Mises diagonal property is valid. Analytically, the determination of the metric coefficients is shown to depend upon the solution of a nonlinear second order partial differential equation. Special solutions are studied. (Received September 28, 1942.)

327. W. R. Foote, Hillel Poritsky, and J. J. Slade: *The motion of a rotor with unequal shaft flexibilities mounted in bearings of unequal flexibility.*

In this paper is discussed the motion of a rotor possessing different flexibilities in two mutually perpendicular directions and mounted in bearings which likewise possess different flexibilities. The equations of motion of such a rotor revolving at constant speed form a fourth order linear system with periodic coefficients. The passage from stable to unstable solution is investigated in detail. For a fourth order system, unlike the second order Mathew equation, this passage may (and does) occur not only through periodic solutions when the characteristic exponent vanishes (or is equal to i) but also by having some of the characteristic exponents equal to each other. What are believed to be new and more convenient methods of calculation are developed. Some of these readily carry over to any order system with periodic coefficients which are trigonometric polynomials. (Received August 4, 1942.)

328. R. P. Isaacs: *Application of polygenic functions to two-dimensional elasticity.*

Let σ and τ be the tensile and shearing stresses in a two-dimensional body on a lineal element at z and inclined at θ with the real axis. If all internal parts of the body are in equilibrium, $\sigma+i\tau$ is the derivative of a polygenic function g which satisfies $I[G_z]=0$, and the derivative of any such function is a stress distribution in equilibrium. The derivative circle of g is the Mohr circle. $G_{zzz}=0$ is necessary and sufficient for the body to obey Hooke's law. (Received September 12, 1942.)

GEOMETRY

329. Edward Kasner and J. J. DeCicco: *A generalized theory of dynamical trajectories.*

The differential geometry of the dynamical trajectories of positional fields of force has been completely developed in the *Differential-Geometric Aspects of Dynamics*, American Mathematical Society Colloquium Publications, vol. 3, 1913 (also Transactions of this Society, vols. 7–9 (1906–1910)). The fields of force, considered heretofore, depend only upon the position of the point. In their new work, the authors begin the study of the geometry of the generalized dynamical trajectories of fields of force which depend not only upon the position of the point but also upon the direction through the point. There are ∞^3 generalized dynamical trajectories. Any such system is completely characterized by the geometric Property I. The complete relationship between the rest trajectories and the lines of force is obtained (including the generalization of the theorem about one-third the curvature). Other systems of curves, such as velocity systems, systems S_k, pressure curves are defined for our generalized fields of force. These three new systems also possess the Property I. Complete geometric discussions of these systems are given. (Received August 24, 1942.)

330. Karl Menger: *Projective generalization of metric geometry.* I.

An F-projective space is called a set P if with each quadruple of elements p, q, r, s no three of which are identical an element (p, q, r, s) of a field F or ∞ is associated and $(p, q, r, s) = (q, p, s, r) = (r, s, p, q)$ and $(p, q, r, s) + (p, r, q, s) = (p, q, r, s) \cdot (p, s, r, q) = 1$. An F-projective space, called $P_1(F)$, is obtained if for the elements of F the cross-ratio is defined as usual. In order that P be imbeddable into $P_1(F)$ it is necessary and sufficient that P satisfy the relation $(p, q, r, s) \cdot (p, s, r, t) = (p, q, r, t)$ and contain no pseudo-degenerate quadruple (four distinct elements with cross-ratio 0). The only non-imbeddable triples and pairs are pseudo-harmonic (that is, have cross-ratio -1). (Received August 3, 1942.)

331. Karl Menger: *Projective generalizations of metric geometry.* II.

P is called an $|F|$-projective space if with each quadruple p, q, r, s no three of whose elements are identical, a pair $(f, -f)$ of elements of F with the sum 0, denoted by $|p, q, r, s|$, is associated and $|p, q, r, s| = |q, p, s, r| = |r, s, p, q|$, $|p, q, r, s| \cdot |p, s, r, q| = |p, q, r, s| \cdot |p, r, s, q| \cdot |p, s, q, r| = 1$, and $1 + (f^3 - g^2)^2 = 2(f^3 + g^2)$ for $f = |p, q, r, s|$ and $g = |p, r, q, s|$. $P_1|F|$ is called the set F if $|p, q, r, s| = |(p, q, r, s)|$. In order that P be imbeddable into $P_1|F|$ it is necessary and sufficient that P satisfy the condition $|p, q, r, s| \cdot |p, s, r, t| = |p, q, r, t|$ and does not contain pseudo-degenerate quadruples or pseudo-harmonic triples or pairs, that is, distinct points with $|p, q, r, s| = |1|$ or not distinct points with $|p, q, r, s| = |2|$. (Received August 3, 1942.)

332. Saly R. R. Struik: *Emmy Noether and Maria Gaetana Agnesi, two outstanding women mathematicians.*

The work and life of these two mathematicians are evaluated and compared. (Received August 6, 1942.)

LOGIC AND FOUNDATIONS

333. A. R. Schweitzer: *On a class of ordered $(n+1)$-ads relevant to the algebra of logic.* III.

The author analyzes the formal sum (now not necessarily reflexive) of $(n+1)$-ads of generalized constituent type into two types of ordered dyads: $\xi\cdot\eta$, multiplicative, and $(\xi)+(\eta)$, additive, where $(\xi)=\xi_1\xi_2\cdots\xi_{n+1}$. Addition is assumed commutative and associative; multiplication is associative but not necessarily commutative. Formal addition is extended to elements ξ by assuming the formal equivalence: $(\alpha_1+\lambda_1)(\alpha_2+\lambda_2)\cdots(\alpha_{n+1}+\lambda_{n+1})=\alpha_1\alpha_2\cdots\alpha_{n+1}+\lambda_1\alpha_2\cdots\alpha_{n+1}+\cdots+\lambda_1\lambda_2\cdots\lambda_{n+1}$. This assumption effects a gradual transition from the author's theory to formal developments based on the distributive property such as Grassmann's extensive algebra, the algebra of logic, Frobenius' calculus of group elements, number systems (fields) and so on. Grassmann's algebra contrasts with the latter in reference to closure properties. These are defined in terms of completeness (extension, invariance, attainment of limit of development) of the fundamental set of elements with reference to adjunction of operations on elements ξ, for example, $\xi\cdot\eta$, $\xi+\eta$. Finally the author discusses applications of restricted distributive properties such as $(x+y)y=xy+y^2$, $(x+I)y=xy+Iy$, where $Iy=y$ and addition and multiplication may or may not be reflexive. Reference is made to a paper reported in this Bulletin, abstract 48-3-135. (Received August 15, 1942.)

334. Saly R. R. Struik: *Axiomatics of affine geometry.*

The axiomatics are developed for two and three dimensions separately. The theorem of Desargues is used as an axiom in two dimensions. (Received August 6, 1942.)

STATISTICS AND PROBABILITY

335. T. N. E. Greville: *Regularity of label-sequences under configuration transformations.*

There is developed a class of transformations on sequences of arbitrary labels in terms of which a wide variety of problems in the theory of probability can be formulated. It is shown that, with mild restrictions on the transformations used and on the measure function assumed on the label-space, almost every label-sequence produces a transform having the frequency distribution expected. The class of transformations considered is shown to include as special cases the four fundamental operations of von Mises: place selection, partition, mixing, and combination. (Received August 4, 1942.)

336. H. B. Mann: *The construction of orthogonal Latin squares.*

A Latin square is based on a group G if in the reduced form its rows form a regular representation of G. A set of orthogonal squares is based on G if all the squares of the set are based on the same representation of G. The mappings $S_1, S_2, \cdots, S_r$ of G onto itself are r-fold complete if every element of G is of the form $X^{s_i+s_{i+1}+\cdots+s_{i+h}}$ for every i and h with $1\leq i\leq r-h$ where X^{s_i} is the image of X under the mapping S_i and $X^{s_i+s_j}=X^{s_i}X^{s_j}$. A set of r orthogonal squares based on G exists if and only if G admits an r-fold complete mapping and vice versa. No $4n+2$ sided Graeco-Latin square based on a group exists. The orthogonal set is constructed by the automorphism method if for every i the mapping $S_1+S_2+\cdots+S_i$ is an automorphism of G. If

c_q is the number of classes of elements of order q in G, then not more than min c_q orthogonal squares can be constructed from G by the automorphism method. (Received September 30, 1942.)

337. Abraham Wald: *On a statistical problem arising in the classification of an individual in one of two groups.*

Let π_1 and π_2 be two p-variate normal populations which have a common covariance matrix. A sample of size N_i is drawn from the population π_i $(i=1, 2)$. Denote by $x_{i\alpha}$ the αth observation on the ith variate in π_1, and by $y_{i\beta}$ the βth observation on the ith variate in π_2. Let z_i $(i=1, \cdots, p)$ be a single observation on the ith variate drawn from a population π where it is known that π is equal either to π_1 or to π_2. The parameters of the populations π_1 and π_2 are assumed to be unknown. It is shown that for testing the hypothesis $\pi=\pi_1$ a proper critical region is given by $U \geqq d$ where $U=\sum\sum s^{ij}z_i(\bar{y}_j-\bar{x}_j)$, $\|s^{ij}\|=\|s_{ij}\|^{-1}$, $s_{ij}=[\sum_\alpha(x_{i\alpha}-\bar{x}_i)(x_{j\alpha}-\bar{x}_j)+\sum_\beta(y_{i\beta}-\bar{y}_i)(y_{j\beta}-\bar{y}_j)]/(N_1+N_2-2)$, $\bar{x}_i=(\sum_\alpha x_{i\alpha})/N_1$, $\bar{y}_i=(\sum_\beta y_{i\beta})/N_2$ and d is a constant. The large sample distribution of U is derived and it is shown that U is a simple function of three angles in the sample space whose exact joint sampling distribution is derived. (Received August 7, 1942.)

338. Jacob Wolfowitz: *On the consistency of a class of non-parametric statistics.*

Let X and Y be two stochastic variables about whose distribution nothing is known except that they are continuous and let it be required to test whether their distribution functions are the same. Let V be the observed sequence of zeros and ones constructed as described elsewhere (Wald and Wolfowitz, Annals of Mathematical Statistics, vol. 11 (1940), p. 148). Suppose that the statistic $S(V)$ used to test the hypothesis is of the form $S(V)=\sum\phi(l_j)$, where l_j is the length of the jth run and $\phi(x)$ a suitable function defined for all positive integral x. The notion of consistency, originated by Fisher for parametric problems, has already been extended to the non-parametric case (loc. cit., p. 153). The author now proves that, subject to reasonable conditions on $\phi(x)$ and statistically unimportant restrictions on the alternatives to the null hypothesis, statistics of the type $S(V)$ are consistent. In particular, a statistic discussed by the author (Annals of Mathematical Statistics, vol. 12 (1942)) and for which $\phi(x)=\log(x^x/x!)$ belongs to the class covered by the theorem. (Received August 7, 1942.)

Topology

339. O. G. Harrold: *A higher dimensional analogue of a theorem of plane topology.*

Since the carriers of a Vietoris cycle may have a dimensionality far removed from that of the cycle, it is of interest to determine a class of spaces for which the bounding cycles have membranes of dimensionality exceeding that of the cycle by unity. An example is known of an $1c^1$ carrying an essential 1-cycle which has a 1-dimensional carrier but bounds only on a 3-dimensional set. A similar example is constructed in the Euclidean space E_6. That such cannot happen in certain Euclidean spaces is indicated by the following theorem, which is a generalization of a known result for $n=0$. Let X be a compact $1c^n$ subset of E_{n+2}. Denote by F the frontier of X relative to E_{n+2}. There exists in X a compact subset X_0 which is $1c^n$ such that $X_0 \supset F$ and $\dim X_0 \leqq n+1$. (Received August 4, 1942.)

340. F. B. Jones: *A characterization of the sphere.*

Let M denote a compact metric continuous curve such that (1) the sum of no two points separates M and (2) every simple closed curve in M separates M. Kline has raised the question as to whether M is a simple closed surface or not. The main result of this paper shows that it is if M contains no skew curve of type 1. (Received August 5, 1942.)

341. Saunders MacLane and Samuel Eilenberg: *Functions of groups as generalized tensors.*

Let Hom $\{G, H\}$ denote the group of all homomorphisms θ of the group G into the group H, and let α_1, β_1 be fixed homomorphisms of G into G_1, H into H_1, respectively. Then $\theta \in \text{Hom}\ \{G_1, H\}$ "induces" $\beta_1\theta\alpha_1 \in \text{Hom}\ \{G, H_1\}$, and the correspondence $\theta \to \beta_1\theta\alpha_1$ is itself a homomorphism $T(\alpha_1, \beta_1)$ of the first group (of θ's) into the second group Hom $\{G, H_1\}$. This function T has the property $T(\alpha_2\alpha_1, \beta_2\beta_1) = T(\alpha_1, \beta_2)T(\alpha_2, \beta_1)$. A function of two groups G, H, like the function "Hom," associated with such a function T of mappings of these groups, is called a tensor "covariant" in H, "contravariant" in G. The intuitive notion of a "natural isomorphism" between two groups can be defined exactly as an isomorphism between tensor functions of groups. This paper extends the same concepts from groups to topological spaces, to rings, to complexes, and to other types of mathematical systems. A number of general properties of tensors and their isomorphisms are established. (Received August 3, 1942.)

342. R. L. Moore: *Concerning a continuum and its boundary.*

If H, K and T are subsets of the continuum M then T is said to shield H from K in M provided it contains no point of K but intersects every connected subset of M which intersects both H and K. Among other things, it is shown that if, in a metric space, A and B are points of β, the boundary of the compact continuum M, and for each component x of $M-\beta$ there is a continuum β_x lying in β and shielding $A+B$ from x then A and B belong to the same component of β. (Received September 10, 1942.)

343. R. L. Moore: *Concerning intersecting continua.*

One of the results obtained is the theorem that if, in a metric space, β is the boundary of the compact continuum M and K is a compact continuum intersecting M and, for every component D of $M-\beta$ and component E of the complement of $\bar{D}$, the boundary of E is a connected subset of β, then $M \cdot K$ is a subset of a component of $M \cdot K+\beta$. (Received September 10, 1942.)

344. C. N. Reynolds: *A calculus of finite topological differences with application to the four color problem.*

A topological derivative of any function with respect to a well defined transformation of the neighborhood of an arbitrary boundary of a map is defined and is so applied to the four color problem as to reduce it to an equivalent problem in the theory of functions of a real variable. (Received August 4, 1942.)

345. A. D. Wallace: *A characterization of unicoherence.*

Let S be a Peano space. If $S=A+B+C$ is a decomposition into continua such that $AB, BC, CA \neq 0$ then assuming S to be unicoherent it is readily seen that $ABC \neq 0$. For $S=A+(B+C)$ so that, since $B+C$ is a continuum, so also is $A(B+C)=AB+AC$. The summands in the right member being non-void and closed it follows that they must intersect. The converse of this is also true and the proof is contained implicitly in the proof of Theorem 1 of *Monotone coverings and monotone transformations*, Duke Mathematical Journal, vol. 6 (1940), p. 32. (Received September 15, 1942.)

346. G. T. Whyburn: *Homotopy classes of mappings into the circle.*

In this paper it is shown primarily that any continuous mapping of a locally connected continuum X into a circle S is homotopic to a mapping f which admits factorization into the form $f=f_2f_1$, where f_1 is monotone and f_2 is interior and light (thus f is quasi-monotone). Used in the proof of this main theorem are auxiliary results to the effect that (1) any continuous mapping f of X into S is interior at every $x \in X$ which is a component of $f^{-1}f(x)$ and which is not contained in arbitrarily small neighborhoods U with $F(U) \subset f^{-1}(y)$ for some $y \in S-f(x)$; (2) a light mapping f of X into S is interior if and only if, for any $y \in S$, every component of $X-f^{-1}(y)$ maps onto $S-y$; (3) any non-alternating mapping f of X into S factors into the form $f=f_2f_1$, where f_1 is monotone and f_2 is non-alternating, interior and light; thus any non-alternating light mapping of X into S is interior; (4) any continuous mapping f of X into S such that, for any $y \in S$, each component of $X-f^{-1}(Y)$ maps onto $S-y$ is quasi-monotone. (Received August 5, 1942.)

347. J. W. T. Youngs: *A note on separation axioms and their application in the theory of a locally connected topological space.*

This note is in the nature of a sequel to a paper by G. E. Albert and the author (*The structure of locally connected topological spaces*, Transactions of this Society, vol. 51 (1942), pp. 637–654). A new separation axiom is introduced. T: If $x \cdot y=0$, then $\bar{x} \cdot \bar{y}=0$. In strength this lies between the T_0 and T_1 axioms. Using the terminology of the paper mentioned above it is shown that the class of locally connected T-spaces is hereditary. (Received August 3, 1942.)

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